Online Computing of Non-Stationary Distributions Velocity Fields by an Accuracy Controlled Growing Neural Gas

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Motivation
Cognition is discrete

Anchoring symbols
Motivation
Cognition is discrete

Anchoring symbols
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Cognition is discrete

Anchoring symbols
Motivation

Cognition is discrete

ANCHORING SYMBOLS

CHAIR
Motivation

Cognition is discrete

Motivation

Cognition is discrete

Anchoring symbols

CHAIR
Motivation
Situatedness

The information is not always in the signal

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Motivation

Situatedness

Can you see the cow?
Motivation

Situatedness
Object recognition from images cannot be entirely bottom-up. Perception, in general terms, involves a situated agent with active sensing skills.
Motivation
Structure from signal

Bottom-up approaches may be worth it!

A cycle with a falling tail on the right!

demo
Motivation

Structure from signal

A dynamical approach

GNG needs to be stabilized, the number of prototypes (i.e. the quantification accuracy) needs to be controlled... once defined.
Notations

Rejection sampling

At each time

- Let $X$ be a bounded input space.
- $\xi \in X$ is an input sample.
Computation of $S_p^N$

1: $S_p^N \leftarrow \emptyset$
2: for $i = 1$ to $N$ do
3: \[ \xi \sim \mathcal{U}_X, \ u \sim \mathcal{U}_{[0,1]} \]
4: if $u < p(\xi)$ then
5: \[ S_p^N \leftarrow S_p^N \cup \{\xi\} \]
6: end if
7: end for
8: return $S_p^N$
Notations

Rejection sampling

Computation of $S^N_p$

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7: end for
8: return $S^N_p$

- $|S^N_p| < N$
- $N$ matters
Notations

Prototypes and distortion

- $\Omega^*_\kappa = \{\omega_1, \omega_2, \cdots, \omega_\kappa\} \in X^{\kappa}$.

- $\omega^*_\xi = \arg\min_{\omega \in \Omega^*_\kappa} \|\xi - \omega\|^2$

- The distortion by $\Omega^*_\kappa$ on $S$

$$\mathcal{E}^S_{\Omega^*_\kappa} = \frac{1}{|S|} \sum_{\xi \in S} \|\xi - \omega^*_\xi\|^2$$

Vector quantization

$$\text{find } \Omega^*, S = \arg\min_{\Omega^*_\kappa \in X^{\kappa}} \mathcal{E}^S_{\Omega^*_\kappa}$$
Notations
Prototypes and distortion

- \( \Omega_{\kappa} = \{\omega_1, \omega_2, \cdots, \omega_\kappa\} \in X^{|\kappa|} \).
- \( \omega^*_\xi = \arg\min_{\omega \in \Omega_{\kappa}} \|\xi - \omega\|^2 \).
- The distortion by \( \Omega_{\kappa} \) on \( S \)
  \[ E^S_{\Omega_{\kappa}} = \frac{1}{|S|} \sum_{\xi \in S} \|\xi - \omega^*_\xi\|^2 \]

Vector quantization
find \( \Omega^*_{\kappa, S} = \arg\min_{\Omega_{\kappa} \in X^{|\kappa|}} E^S_{\Omega_{\kappa}} \)
Motivation

Notations

Controlling the quantization accuracy

Velocity field

Conclusion

Rejection sampling

Prototypes and distortion

Property for $\Omega^*, S$

Notations

Prototypes and distortion

- $\Omega_\kappa = \{\omega_1, \omega_2, \ldots, \omega_\kappa\} \in X^{\kappa}$.
- $\omega^*_\xi = \arg\min_{\omega \in \Omega_\kappa} \|\xi - \omega\|^2$
- The distortion by $\Omega_\kappa$ on $S$
  \[ \mathcal{E}^S_{\Omega_\kappa} = \frac{1}{|S|} \sum_{\xi \in S} \|\xi - \omega^*_\xi\|^2 \]

Vector quantization

$\text{find } \Omega^*_\kappa, S = \arg\min_{\Omega_\kappa \in X^{\kappa}} \mathcal{E}^S_{\Omega_\kappa}$
Motivation
Notations
Controlling the quantization accuracy
Velocity field
Conclusion

Notations
Prototypes and distortion

\[ \Omega_\kappa = \{\omega_1, \omega_2, \ldots, \omega_\kappa\} \in X^{|\kappa|}. \]
\[ \omega_\xi^* = \arg\min_{\omega \in \Omega_\kappa} \|\xi - \omega\|^2 \]

The distortion by \( \Omega_\kappa \) on \( S \)
\[ \mathcal{E}_{\Omega_\kappa}^S = \frac{1}{|S|} \sum_{\xi \in S} \|\xi - \omega_\xi^*\|^2 \]

Vector quantization

\[ \text{find } \Omega_{\kappa,S}^* = \arg\min_{\Omega_\kappa \in X^{|\kappa|}} \mathcal{E}_{\Omega_\kappa}^S \]
Notations

Prototypes and distortion

- \( \Omega_\kappa = \{\omega_1, \omega_2, \cdots, \omega_\kappa\} \in X^{\kappa} \).
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\( \text{find } \Omega_{\kappa,S}^* = \arg\min_{\Omega_\kappa \in X^{\kappa}} \mathcal{E}_{\Omega_\kappa}^S \)
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Prototypes and distortion

- \( \Omega_\kappa = \{\omega_1, \omega_2, \cdots \omega_\kappa\} \in X^{\kappa} \).
- \( \omega^*_\xi = \arg\min_{\omega \in \Omega_\kappa} ||\xi - \omega||^2 \)
- The distortion by \( \Omega_\kappa \) on \( S \)
  \( \mathcal{E}^S_{\Omega_\kappa} = \frac{1}{|S|} \sum_{\xi \in S} ||\xi - \omega^*_\xi||^2 \)

Vector quantization

find \( \Omega^*_\kappa, S = \arg\min_{\Omega_\kappa \in X^{\kappa}} \mathcal{E}^S_{\Omega_\kappa} \)
Notations

Prototypes and distortion

- \( \Omega_\kappa = \{\omega_1, \omega_2, \cdots, \omega_\kappa\} \in X^{\kappa} \).
- \( \omega_\xi^* = \arg\min_{\omega \in \Omega_\kappa} \|\xi - \omega\|^2 \)
- The distortion by \( \Omega_\kappa \) on \( S \)
  \[
  \mathcal{E}_{\Omega_\kappa}^S = \frac{1}{|S|} \sum_{\omega \in \Omega_\kappa} \sum_{\xi \in V_\omega^S} \|\xi - \omega\|^2
  \]

Vector quantization

find \( \Omega^*_\kappa, S = \arg\min_{\Omega_\kappa \in X^{\kappa}} \mathcal{E}_{\Omega_\kappa}^S \)
Motivation

Notations

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Prototypes and distortion

Property for $\Omega^*_\kappa, S$

Notations

Prototypes and distortion

- $\Omega_\kappa = \{\omega_1, \omega_2, \cdots, \omega_\kappa\} \in \mathcal{X}^{\kappa}$.
- $\omega^*_\xi = \arg\min_{\omega \in \Omega_\kappa} \|\xi - \omega\|^2$
- The distortion by $\Omega_\kappa$ on $S$
  \[ \mathcal{E}^S_{\Omega_\kappa} = \frac{1}{|S|} \sum_{\omega \in \Omega_\kappa} \sum_{\xi \in V^S_\omega} \|\xi - \omega\|^2 \]

Vector quantization

\[
\text{find } \Omega^*_\kappa, S = \arg\min_{\Omega_\kappa \in \mathcal{X}^{\kappa}} \mathcal{E}^S_{\Omega_\kappa}
\]
## Notations

**Prototypes and distortion**

- $\Omega_\kappa = \{\omega_1, \omega_2, \cdots, \omega_\kappa\} \in X^{|\kappa|}$.
- $\omega^*_\xi = \arg\min_{\omega \in \Omega_\kappa} \|\xi - \omega\|^2$
- The distortion by $\Omega_\kappa$ on $S$
  \[ E^S_{\Omega_\kappa} = \frac{1}{|S|} \sum_{\omega \in \Omega_\kappa} \sum_{\xi \in V^S_\omega} \|\xi - \omega\|^2 \]

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### Vector quantization

Find $\Omega^*_\kappa, S = \arg\min_{\Omega_\kappa \in X^{|\kappa|}} E^S_{\Omega_\kappa}$
**Motivation**

**Notations**

**Controlling the quantization accuracy**

**Velocity field**

**Conclusion**

**Rejection sampling**

**Prototypes and distortion**

**Property for \( \Omega_{\kappa, \mathcal{S}}^* \)**

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**Notations**

Property for \( \Omega_{\kappa, \mathcal{S}}^* \)

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The Voronoi distortion \( \mathcal{V}_\omega^S \) is equally shared
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The Voronoï distortion $V^S_\omega$ is equally shared.
Notations

Property for $\Omega^*_\kappa, S$

The Voronoï distortion $\mathcal{V}_\omega^S$ is equally shared

Prototypes and at most 5000 samples

Voronoi error map

Voronoi error histogram
The Voronoi distortion $\mathcal{V}_\omega^S$ is equally shared.

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Online Computing of Non-Stationary Distributions Velocity Field
The Voronoï distortion histogram contains **density independent** information about the quantization accuracy.
Controlling the quantization accuracy

The influence of $N$ (in $S_p^N$)

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$$v_\omega^S = \sum_{\xi \in V_\omega^S} \|\xi - \omega\|^2$$
Controlling the quantization accuracy

The influence of $N$ (in $S_p^N$)

The influence of $N$ (in $S_p^N$)

\[ V^S_\omega = \sum_{\xi \in V^S_\omega} ||\xi - \omega||^2 \]

- $|S_p^{k.N}| \approx k.|S_p^N|$
- $\forall \omega \in \Omega_\kappa, \ V^S_{\omega}^{k.N} \approx k. V^S_{\omega}^N$
- $\forall \omega \in \Omega_\kappa, \ V^S_{\omega}^{k.N} \approx k. V^S_{\omega}^N$

$\Rightarrow \forall \omega \in \Omega_\kappa^*, S, \ V^S_{\omega}^N \approx T.N$
Controlling the quantization accuracy

The influence of $N$ (in $S^N_P$)

\[ \nu^S_\omega = \sum_{\xi \in V^S_\omega} \| \xi - \omega \|^2 \]

- $|S^{k,N}_P| \approx k \cdot |S^N_P|$  
- $\forall \omega \in \Omega_k, |V^{S^{k,N}}_\omega| \approx k \cdot |V^{S^N}_\omega|$  
  - $\forall \omega \in \Omega_k, V^{S^{k,N}}_\omega \approx k \cdot V^{S^N}_\omega$  
  - $\forall \omega \in \Omega^*_k, S, V^{S^N}_\omega \approx T \cdot N$
Controlling the quantization accuracy

The influence of $N$ (in $S^N_p$)

Interpretation of $T$

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Online Computing of Non-Stationary Distributions Velocity Field
Controlling the quantization accuracy

The influence of $N$ (in $S^N_p$)

$$\forall \omega \in \Omega_{\kappa}, V^{S^N_p}_{\omega} \approx k. V^{S^N_p}_{\omega}$$

$$\Rightarrow \forall \omega \in \Omega^*_{\kappa,S}, V^{S^N_p}_{\omega} \approx T.N$$

$$V^S_\omega = \sum_{\xi \in V^S_\omega} ||\xi - \omega||^2$$
Controlling the quantization accuracy

VQ-T

1: Sample $S = S_p^N$, $\kappa = 1$.
2: repeat
3: Compute $\Omega^{*}_{\kappa, S}$ and $\mathcal{H} = \text{histo } \nu^{S}_{\omega}$.
4: Compute $[a, b] = \text{SCI}_\delta (\mathcal{H})$
5: if $T.N < a$ then
6: $\kappa \leftarrow \kappa - 1$
7: else if $T.N > b$ then
8: $\kappa \leftarrow \kappa + 1$
9: end if
10: until $T.N \in [a, b]$
Controlling the quantization accuracy

VQ-T

1: Sample $S = S_p^N$, $\kappa = 1$.
2: repeat
3: Compute $\Omega^*_{\kappa, S}$ and $H = \text{histo} \, \nu^S_{\Omega^*_{\kappa, S}}$.
4: Compute $[a, b] = \text{SCI}_\delta (H)$
5: if $T.N < a$ then
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Controlling the quantization accuracy

VQ-T

1: Sample $S = S_p^N$, $\kappa = 1$.
2: repeat
3: Compute $\Omega_{\kappa,S}^*$ and $\mathcal{H} = \text{histo} \, \mathcal{V}_S^\omega$.
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Controlling the quantization accuracy

VQ-T

1: Sample \( S = S_p^N, \kappa = 1. \)
2: repeat
3: Compute \( \Omega^{\star}_{\kappa, S} \) and \( \mathcal{H} = \text{histo} \left( \mathcal{V}^S_{\omega \in \Omega^{\star}_{\kappa, S}} \right) \).
4: Compute \([a, b] = \text{SCI}_\delta (\mathcal{H})\).
5: if \( T.N < a \) then
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Controlling the quantization accuracy

VQ-T

1: Sample $S = S^N_p, \kappa = 1$.
2: repeat
3: Compute $\Omega^*_{\kappa, S}$ and $\mathcal{H} = \text{histo } \mathcal{V}^S_\omega$.
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Controlling the quantization accuracy

**VQ-T**

\[ T.N \text{ and } \text{histo } \gamma^S_{\omega} \text{ gives numerical arguments for choosing } \kappa \text{ in any vector quantization technique.} \]
Controlling the quantization accuracy

Interpretation of $T$

\[
\nu^S_\omega = \sum_{\xi \in V^S_\omega} \|\xi - \omega\|^2
\]

- Let us take $\forall \xi \in X, \ p(\xi) = 1$
- \[
\frac{1}{|\nu_\omega|} \int_{\xi \in \nu_\omega} \|\xi - \omega\|^2 d\xi = d_\omega \approx \frac{\nu^S_\omega}{|V^S_\omega|}
\]
- \[
\nu^S_\omega \approx |V^S_\omega|.d_\omega \approx \frac{N}{d_\omega}
\]
- \[
\nu^S_\omega \approx T.N - T = T^S
\]
Controlling the quantization accuracy

Interpretation of $T$

\[ \mathcal{V}_\omega^S = \sum_{\xi \in \mathcal{V}_\omega^S} \|\xi - \omega\|^2 \]

- Let us take $\forall \xi \in X$, $p(\xi) = 1$
- \[ \frac{1}{|\nu_\omega|} \int_{\xi \in \nu_\omega} \|\xi - \omega\|^2 d\xi = d_\omega \approx \frac{\mathcal{V}_\omega^S}{|\mathcal{V}_\omega^S|} \]
- \[ \mathcal{V}_\omega^S \approx |\mathcal{V}_\omega^S| \cdot d_\omega \approx \frac{N}{\kappa^2} \cdot d_\omega \]
- \[ \mathcal{V}_\omega^S \approx T.N \Rightarrow T = \frac{d_\omega}{\kappa^2} \]
Controlling the quantization accuracy

Interpretation of $T$

\[ \mathcal{V}_\omega^S = \sum_{\xi \in \mathcal{V}_\omega^S} \|\xi - \omega\|^2 \]

- Let us take $\forall \xi \in \mathcal{X}, \ p(\xi) = 1$
- \[ \frac{1}{|\nu_\omega|} \int_{\xi \in \nu_\omega} \|\xi - \omega\|^2 \, d\xi = d_\omega \approx \frac{\mathcal{V}_\omega^S}{|\mathcal{V}_\omega^S|} \]
- \[ \mathcal{V}_\omega^S \approx |V_\omega^S| \cdot d_\omega \approx \frac{N}{\kappa^*} \cdot d_\omega \]
- \[ \mathcal{V}_\omega^S \approx T \cdot N \Rightarrow T = \frac{d_\omega}{\kappa^*} \]
Controlling the quantization accuracy

Interpretation of $T$

\[ V^S_\omega = \sum_{\xi \in V^S_\omega} \|\xi - \omega\|^2 \]

- Let us take $\forall \xi \in X, \ p(\xi) = 1$
- \[ \frac{1}{|\nu_\omega|} \int_{\xi \in \nu_\omega} \|\xi - \omega\|^2 d\xi = d_\omega \approx \frac{V^S_\omega}{|V^S_\omega|} \]
- \[ V^S_\omega \approx |V^S_\omega| \cdot d_\omega \approx \frac{N}{\kappa^*} \cdot d_\omega \]
- \[ V^S_\omega \approx T \cdot N \Rightarrow T = \frac{d_\omega}{\kappa^*} \]
Controlling the quantization accuracy

Interpretation of $T$

\[ \nu_\omega^S = \sum_{\xi \in V_\omega^S} \|\xi - \omega\|^2 \]

- Let us take $\forall \xi \in X$, $p(\xi) = 1$
- \[ \frac{1}{|\nu_\omega|} \int_{\xi \in \nu_\omega} \|\xi - \omega\|^2 d\xi = d_\omega \approx \frac{\nu_\omega^S}{|V_\omega^S|} \]
- \[ \nu_\omega^S \approx |V_\omega^S| \cdot d_\omega \approx \frac{N}{\kappa^*} \cdot d_\omega \]
- \[ \nu_\omega^S \approx T \cdot N \Rightarrow T = \frac{d_\omega}{\kappa^*} \]
Controlling the quantization accuracy

Interpretation of $T$

\[ T = \frac{d_\omega}{\kappa^*} \]

- $|\nu_\omega| \approx |X|/\kappa^* \approx 1/\kappa^*$
- $d_\omega = \frac{1}{|\nu_\omega|} \int_{\xi \in \nu_\omega} \|\xi - \omega\|^2 d\xi = 1/2\pi \kappa^*$
- $T = 1/2\pi \kappa^*$

Numerical application
Numerical application

\[ T = \frac{d_\omega}{\kappa^*} \]

- \( |\nu_\omega| \approx \frac{|X|}{\kappa^*} \approx \frac{1}{\kappa^*} \)
- \( d_\omega = \frac{1}{|\nu_\omega|} \int_{\xi \in \nu_\omega} \|\xi - \omega\|^2 d\xi = \frac{1}{2\pi \kappa^*} \)
- \( T = \frac{1}{2\pi \kappa^*} \)
Controlling the quantization accuracy

Interpretation of $T$

\[ T = \frac{d_\omega}{\kappa^*} \]

- $|\nu_\omega| \approx |X|/\kappa^* \approx 1/\kappa^*$
- $d_\omega = \frac{1}{|\nu_\omega|} \int_{\xi \in \nu_\omega} \|\xi - \omega\|^2 \, d\xi = 1/2\pi\kappa^*$
- $T = 1/2\pi\kappa^{*2}$
Controlling the quantization accuracy

Interpretation of $T$

$$T = \frac{d_\omega}{\kappa^*}$$

- $|\nu_\omega| \approx |X|/\kappa^* \approx 1/\kappa^*$
- $d_\omega = \frac{1}{|\nu_\omega|} \int_{\xi \in \nu_\omega} \|\xi - \omega\|^2 d\xi = 1/2\pi\kappa^*$
- $T = 1/2\pi\kappa^*$

Controlling the quantization accuracy

Interpretation of $T$

$$T = \frac{d_\omega}{\kappa^*}$$

- $|\nu_\omega| \approx |X|/\kappa^* \approx 1/\kappa^*$
- $d_\omega = \frac{1}{|\nu_\omega|} \int_{\xi \in \nu_\omega} \parallel \xi - \omega \parallel^2 d\xi = 1/2\pi\kappa^*$
- $T = 1/2\pi\kappa^*^2$
Controlling the quantization accuracy

Interpretation of $T$

$T = \frac{1}{2\pi}\kappa^2$, $\kappa^* = 500$, $N = 50000 \Rightarrow T.N \approx 0.0318$, $\kappa = 500$
Controlling the quantization accuracy

Numerical application

\[ T = \frac{1}{2\pi\kappa^*^2}, \quad \kappa^* = 500, \quad N = 50000 \Rightarrow T.N \approx 0.0318, \quad \kappa = 343 \]
Controlling the quantization accuracy

Interpretation of $T$

$T$ is both a **quantitative** and a **qualitative** measure of the vector quantization accuracy, $N$ can be chosen **arbitrarily** without affecting the quantization quality.
Controlling the quantization accuracy

GNG-T

Beyond Growing Neural Gas
Controlling the quantization accuracy

**GNG-T**

Beyond Growing Neural Gas

Controlling the number $\kappa$ of prototypes

Using $\text{mean} \nu_{\omega}^S$, where $\omega \in \Omega_{\kappa, S}^*$

![Graphs showing the evolution of number of vertices and evolution criterion over epochs.](image-url)
Controlling the quantization accuracy

**GNG-T**

Beyond Growing Neural Gas

**Controlling the number \( \kappa \) of prototypes**

Using \( SCI_\delta \left( \text{histo} \ V^S_\omega \right) \)

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**Motivation**

**Notations**

**Controlling the quantization accuracy**

**Velocity field**

**Conclusion**

The influence of \( N \) (in \( S^N \))

VQ-T

Interpretation of T

GNG-T
Velocity field

An ill-posed problem

- Contour modification...
  - ... is ambiguous.
  - Need for a structural constraint.

Optical flow

- Optimize both the fitting to the contour and some smoothness criterion.
- The differential approach requires a high framerate.
**Velocity field**

An ill-posed problem

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Velocity field
An ill-posed problem

An ill-posed problem
Structural smoothing
Video application

The aperture problem

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Optical flow

Optimize both the fitting to the contour and some smoothness criterion.
The differential approach requires a high framerate.
A SOM is not necessarily squared.
- When input changes...
- ... the whole SOM drifts (demo).
- Prototypes velocity field.

The trick
- Disabling evolution: GNG → SOM.
- Done at each frame transition.
Self-organizing map can do the job!

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The trick

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Velocity field
Structural smoothing

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The trick

- Disabling evolution: GNG $\rightarrow$ SOM.
- Done at each frame transition.
The trick

- Disabling evolution: GNG $\rightarrow$ SOM.
- Done at each frame transition.
Adaptive $N$ for $S_p^N$

\[
\alpha = \min\left(\text{nb\_samples}, \text{max\_samples}\right)/\text{nb\_samples}
\]

\[
N = \alpha \times \text{width} \times \text{height}
\]

\[
\text{max\_samples} = 500
\]
Velocity field

Video application

Adaptive $N$ for $S_p^N$

$$\alpha = \min(nb\_samples, \max\_samples)/nb\_samples$$

$$N = \alpha \times \text{width} \times \text{height}$$

$$\max\_samples = 2500$$
Adaptive $N$ for $S_p^N$

$$\alpha = \min(n_{\text{samples}}, \text{max\_samples})/n_{\text{samples}}$$

$$N = \alpha \times \text{width} \times \text{height}$$
Conclusion

To sum up

- GNG-T → bottom-up structuring.
- Modeling dynamical data, $S_p^N$
- $T$ → accuracy understood and controlled.
- GNG-T stabilization →
  - reliable graph on static data.
  - SOM-based velocity field retrieval.
  - robustness to low framerates.
  - Adjustment of time consumption.
- open-source fast C++ implementation is provided.
Conclusion

To sum up

- GNG-T $\rightarrow$ bottom-up structuring.
- Modeling dynamical data, $S_p^N$
- $T \rightarrow$ accuracy understood and controlled.
- GNG-T stabilization $\rightarrow$
  - reliable graph on static data.
  - SOM-based velocity field retrieval.
  - robustness to low framerates.
  - Adjustment of time consumption.
- open-source fast C++ implementation is provided.
The approach is not dedicated to color-based pixel detection. Other incremental algorithms can be controlled. Collaborations...